THE UNIVERSITY OF QUEENSLAND AUSTRALIA

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UQ-CCSG Annual Research Review

Centre for Coal Seam Gas

2018

Adaptive mesh refinement for gravity forward modeling

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1. Gravity forward modeling (GFM)

Gravity inversion is a technique to produce a subsurface model of the earth's subsurface geological structure. The earth's gravitational field is measured at specific locations on the earth's surface or in the sky. It has many uses, including to monitor the change of groundwater, estimate the depth of sedimentary basins and in mineral exploration.

Gravity forward modeling (GFM) is the cornerstone of gravity inversion. It calculates the gravitational effect from a known density distribution. Mathematically, GFM is a Poisson's equation with appropriate boundary conditions, i.e.

$$\nabla^2 \varphi = -4\pi G \rho,$$

where φ, G and ρ are gravitational potential, Newton's gravitational constant and density distribution respectively. Boundary condition in my case is homogeneous potential on part of boundary and homogeneous normal gravity field on the rest.

2. Numerical modeling

Normally, the GFM domain includes irregular topography and small-scale complex interfaces between different densities.

A solution is introducing a method which combines the finite element method (FEM) and adaptive mesh refinement (AMR). AMR not only helps to discretize the domain accurately, but also to modify mesh automatically during computation to obtain an optimal balance between accuracy and costs.

Hexahedral mesh is used because of its ease of generation and its ability to deliver more accurate approximation than tetrahedral mesh. Though hexahedral mesh is not as good when it comes to approximating complex geometries, this shortfall can be easily covered by using AMR. This allows the use of finer elements around topography and complex interfaces. A parallel adaptive mesh refinement library, p4est, is used to facilitate mesh generation and refinement, and distribute elements equally for parallel computing.

3. Domain decomposition

In computation the GFM domain needs to be decomposed into a reference model and residual model. The gravitational effect of the reference model can be easily calculated due to its simple structure, while the residual model is used to model the gravitational effect of any density anomalies.

Domain decomposition is needed, otherwise any gravitational effect due to density anomalies is indiscernible.

The following figure is an example of domain decomposition.

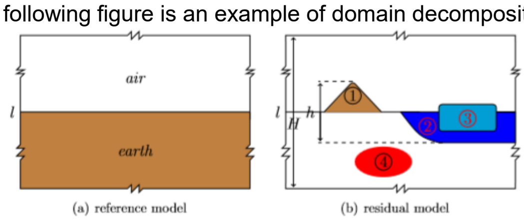

Figure 1: A 2D schematic illustration of computational domain: (a) reference model; (b) residual model with topography (eg. mountain, ocean and iceburg) and subsurface density anomaly.

4. Verification

The current solution method to GFM is validated with a cubic anomaly model, where a cubic anomaly with density $\rho = 2000\ kg/m^3$ and size $L_a = 2^{17}$ m in the center is surrounded by a zero-density padding layer. The merit of this simple validation model is that its analytic gravity field is obtainable thanks to its simple structure.

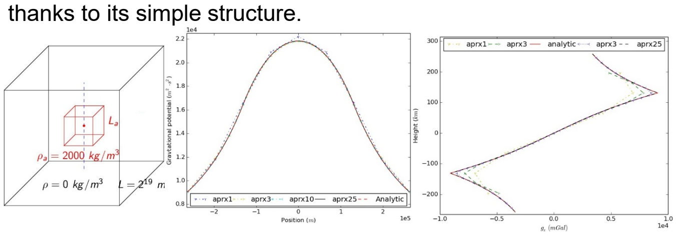

Figure 2: Cubic Anomaly Model (left), comparison of exact and approximated solutions on the central dashed line (middle (gravity potential) and right (gravity field)).

5. North Queensland coast

DEM files providing surface elevation and bedrock elevation data was used to construct the topography for a costal area of north Queensland. The study area was set to a cubic domain of 20 km thick and 549.45 km long in both latitude and longitude. More than half ($5/8$) of the total domain is below mean sea level.

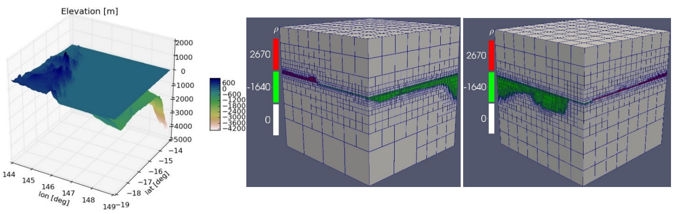

Figure 3: Topography (left), initial mesh and density distribution (middle and right).

n_r	N	e_m	e_v
1	96601	0.00411769	4.19819e-05
2	97301	0.00196696	2.77621e-05
3	100857	0.000936291	1.69646e-05
4	104098	0.000755154	1.49197e-05
5	104224	0.000563673	1.48588e-05

The above table is a summary of error estimations during the adaptive process, where $n_r$and N are refine level and element number, e_m is the largest element-wise error estimation (θ_k) and the average element-wise error estimation is:

$$e_v = \sqrt{\frac{1}{N}\sum_1^N \theta_k^2}$$

The largest element-wise error estimation e_m decreases faster than e_v does, and this is consistent with our adaptive strategy.

Acknowledgement

This research has been conducted with the support of The University of Queensland Centre for Coal Seam Gas (UQ-CCSG).

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